Velocity and frictional effects of porous elliptical plates lubricated with couple stress fluid considering the effects of slip velocity

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Abstract- The analysis of the squeeze film between porous elliptic plates is extended to include the effect of velocity slip at the fluid and the porous material interface. On the basis of Stokes micro continuum theory of couple stress fluid, a modified Reynolds equation is derived governing the fluid velocity and pressure. The present study is to analyse the effect of velocity and friction, considering slip velocity.
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Keywords: coefficient of friction, couple stress fluid, Elliptic plates, Friction, velocity slip.

1. Introduction

The squeeze film behavior arises whenever two lubricated surfaces approach each other with normal velocity. The viscous fluid which is present between the two surfaces cannot be instantaneously squeezed out. It takes some time for these surfaces to come into contact. Hence the viscous lubricant has a resistance to extrusion, a pressure is built up during that interval and the lubricant film then supports the load.

The Stokes (1966) is the simplest generalization of the classical theory of fluids which allows for polar effects such as the presence of couple stresses, body couples and non-symmetric tensors. Ramanaiah (1979) had analysed squeeze film between finite plates of various shapes lubricated by fluids with couple stress. Hays (1963) had investigated the squeeze film phenomena between approaching rectangular plates. The results published by Bujurke and Jayaraman (1982) proved that the bearing with couple stress as lubricant provides significant load supporting characteristics with longer bearing life. J.R Lin (1997) predicted that the presence of couple stress provides an enhancement in the load carrying capacity and lengthens the response time of the squeeze film in journal bearing. Sudha, Sundarammal and Ramamurthy (2008) have analysed squeeze film characteristics of rectangular, elliptic and porous triangular plates lubricated with couple stress fluid.

Whenever an analysis of squeeze films or porous bearing is made it has been customary to assume that for the flow field outside the porous medium, the tangential velocity components are zero at the surface of the porous material. But this actually gives us only an approximation to the actual solution. Beavers and Joseph [1] proposed an alternate boundary condition which admits that a non zero tangential velocity i.e slip velocity at the permeable wall influences the squeezing action. Sparrow et. al.[2] studied its effect on porous walled squeeze films. Using this slip boundary condition Wu studied the effects of slip velocity for porous annular disc [3] and between porous rectangular plates [4]. Murti [5] considered the problem of a squeeze film between two circular discs. Prakash and Vij [6] considered the squeeze film between porous plates of different shapes to include the effect of velocity slip. These investigations show that the effect of velocity slip further reduces the load capacity and the squeeze film approach time. The study of velocity slip was also considered for various bearing. Murti [7], [8] considered the effect of velocity slip in narrow porous bearings and the same with arbitrary wall thickness. K.C.Patel and J.L.Gupta [9] studied the effects of hydrodynamic lubrication of a porous slider bearing with slip velocity. R.C.Shah and M.V.Bhat analysed porous exponential slider bearing with a ferro fluid considering slip velocity [10].In recent times the field has been extended to include the effect of surface roughness also to the various bearings. N.D.Patel and G.Deheri [11] considered the effect of surface roughness on the performance of a magnetic fluid based parallel plate porous slider bearing with slip velocity. P.S.Rao and S.Agarwal [12] have looked into the effects of surface roughness on porous inclined slider bearing lubricated with couple stress fluid.The arbitrary squeezing of a viscous fluid between elliptic plates has been analysed by Usha and Sridharan[13] .The squeeze film behavior between elliptic plates considering the effects of porosity and surface roughness was considered by Manimegalai and Sundarammal[14].In this paper we calculate the friction developed between the elliptic plates considering the effect of velocity slip.

Nomenclature:

f - Coefficient of friction
F - Friction
h - Fluid film thickness
k - Permeability
l - Couple stress parameter
P - Pressure in the film region
P - Pressure gradient
p^* - Pressure in the porous region

s - Slip parameter
u, v, w - Components of fluid velocity in the film regions
u^*, v^*, w^* - Components of fluid velocity in the porous region
W - Load carrying capacity
x, y, z - Cartesian co-ordinates
α - Slip constant
β - Percolation parameter
δ - Thickness of the porous layer
μ - Viscosity
η - New material constant peculiar to fluid with couple stress
τ - Shear stress

2. Analysis

The squeezing flow of couple stress fluid between two elliptic plates is considered. The upper plate is approaching the lower, stationary porous plate. The gap between the plates is filled with a couple stress fluids.

Under the usual assumptions of hydrodynamic lubrication of thin film, the continuity and momentum equations derived by Stokes for the couple stress fluid takes the form

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0 \tag{1}$$

(1)

$$\mu\frac{\partial^2 u}{\partial z^2}-\eta\frac{\partial^4 u}{\partial z^4}=\frac{\partial P}{\partial x} \tag{2}$$

$$\mu\frac{\partial^2 v}{\partial z^2}-\eta\frac{\partial^4 v}{\partial z^4}=\frac{\partial P}{\partial y} \tag{3}$$

$$\frac{\partial P}{\partial z}=0 \tag{4}$$

The flow of the couple stress fluid in the porous matrix is governed by the modified form of the Darcy law for porous material and is given by

$$u^*=\frac{\text{-k}}{\mu(1\text{-}\beta)}\frac{\partial p^*}{\partial x} \tag{5}$$

$$v^*=\frac{\text{-k}}{\mu(1\text{-}\beta)}\frac{\partial p^*}{\partial y} \tag{6}$$

$$w^*=\frac{\text{-k}}{\mu(1\text{-}\beta)}\frac{\partial p^*}{\partial z} \tag{7}$$

The boundary conditions for the velocity components at z = 0 are

$$\frac{\alpha}{\sqrt{k}}(u-u^*)=\frac{\partial u}{\partial y} \tag{8}$$

$$\frac{\alpha}{\sqrt{k}}(v-v^*)=\frac{\partial v}{\partial y} \tag{9}$$

$$w=-w^* \tag{10}$$

$$\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2 v}{\partial z^2}=0 \tag{11}$$

Equation (8) and (9) are the Beaver Joseph slip boundary conditions for the tangential velocity slip at the porous interface..

Solving (2) with the boundary condition that $u=0$ at $z=0$ and $z=h$ gives

$$u=\frac{\frac{\partial P}{\partial x}}{2\mu}\left\{\left(z^2-zh\right)+2l^2\left[1-\frac{\cosh\left(\frac{2z-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right]\right\} \tag{12}$$

Similarly

$$v=\frac{\frac{\partial P}{\partial y}}{2\mu}\left\{\left(z^2-zh\right)+2l^2\left[1-\frac{\cosh\left(\frac{2z-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right]\right\} \tag{13}$$

The pressure p^* in the porous medium satisfies the Laplace equation

$$\nabla^2 p^*=0 \tag{14}$$

Solving this we get

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}=\frac{12\mu\dot{h}}{f(h,c_1,c_2,l)+12\frac{\delta k}{1-\beta}} \tag{15}$$

where

$$f(h,c_1,c_2,l)=h^3(1+c_2)-6h^2lc_1\tanh\left(\frac{h}{2l}\right)-12l^2\left[h-2l\tanh\left(\frac{h}{2l}\right)\right] \tag{16}$$

$$c_1=\frac{s}{s+h} \tag{17}$$

and

$$c_2=-\frac{3}{(1\text{-}\beta)}\left[\frac{2s^2\alpha^2}{(h^2+sh)}+\frac{s(1-\beta)}{(h+s)}\right] \tag{18}$$

The pressure distribution is obtained by solving equation (14) with the boundary condition P=0 on the boundary of the plates.

$$P=-12\mu\dot{h}\left[f(h,c_1,c_2,l)+\frac{12\delta k}{1-\beta}\right]^{-1}\cdot\frac{a^2b^2}{2(a^2+b^2)}\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right) \tag{19}$$

The load is given by

$$W=-\mu A^2B\dot{h}\left[f(h,c_1,c_2,l)+\frac{12\delta k}{1-\beta}\right]^{-1} \tag{20}$$

Where A is the area of the upper plate and

$$B=\frac{12}{A^2}\iint_A \frac{a^2b^2}{2(a^2+b^2)}\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)dxdy \tag{21}$$

The shear stress along the surface is given by

$$\tau=\mu\left(\frac{\partial u}{\partial y}\right) \tag{22}$$

Substituting for u from equation (12) , we have

$$\tau=\frac{\frac{\partial P}{\partial x}}{2}\left\{(2z-h)-2l\left[\frac{\sinh\left(\frac{2z-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right]\right\} \tag{23}$$

At the upper plate $z=h$ the shear stress is given by

$$\tau=\frac{\frac{\partial P}{\partial x}}{2}\left\{h-2l\tanh\left(\frac{h}{2l}\right)\right\} \tag{24}$$

And at the lower plate $z=0$ the shear stress is given by

$$\tau=\frac{\frac{\partial P}{\partial x}}{2}\left\{-h+2l\tanh\left(\frac{h}{2l}\right)\right\} \tag{25}$$

Integration of shear stress over the surface gives the frictional force F.

$$F=\int\tau dx=\int_0^a\left(\frac{\frac{\partial P}{\partial x}}{2}\left\{h-2l\tanh\left(\frac{h}{2l}\right)\right\}\right)dx \tag{26}$$

The coefficient of friction is then given by

$$\bar{f}=\frac{F}{W} \tag{27}$$

which is calculated using equation (20) and equation (26)

3. Results and Discussion

In this section, we have presented the graphical results of the effect of velocity and friction for various parameters. In figure (1) the variation of the velocity when the height of the fluid film varies are observed for various values of h. It is found that as the film height reduces as the velocity increases. In figure (2) the couple stress parameters effect on the velocity is observed. It shows that as there is an increase in the couple stress parameter l, accordingly the velocity u also increases.It can also be observed that since we have fixed the value of h at 0.4, the velocity becomes zero at that particular point after which it starts increasing. In figure (3) the pressure gradient p and its effect on the velocity u are noted .It is clear that with an increase in the pressure gradient the velocity decreases. In figure (4) the effect of the viscosity is compared with the velocity. The graph shows that the increase in viscosity decreases the pressure. The effect of the friction for various values of the couple stress parameter is noted in figure (5). It shows that as the couple stresss parameter l decreases there is an increase in friction. In figure (6) the effect of friction is compared with various values of the pressure gradient. It can be seen that as the pressure gradient increases the friction also increases.

Table.1: Effects of various parameters on velocity u and friction f

Sl.no	Parameter	Values of parameter	Remarks
1.	h	h=0.4,0.6,0.8,1.0,1.2; μ=0.2;l=0.4; p=1	u increases as h decreases
2.	l	l=0.4,0.5,0.6,0.7,0.8; μ=0.2; p=1;h=0.4	u decreases as l increases
3.	p	p=1,2,3,4,5; μ=0.2;l=0.4;h=0.4	u deceases as p increases
4.	μ	μ=1.1,1.2,1.3,1.4,1.5; l=0.4; p=1;h=0.4	u increases as μ decreases
5.	l	l=0.4,0.5,0.6,0.8; p′=1	f increases as l decreases
6.	p	p=1,2,3,4,5; l=0.4	f increases as p′increases

4. Figures

Fig.1 variation of h on u when μ=0.20, l=0.4, p=1, h=0.4, 0.6, 0.8, 1.0, 1.2

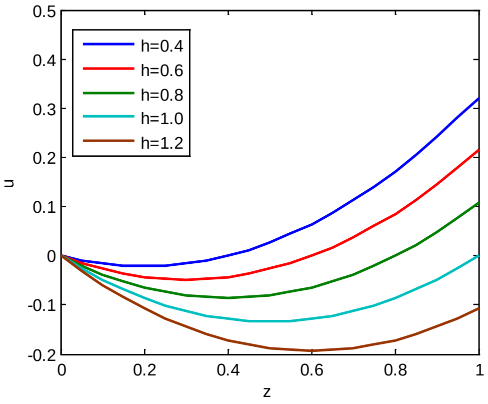


Fig. 2. variation of l on u when μ=0.20,p=1, h=0.4, l=0.4, 0.5, 0.6, 0.7, 0.8

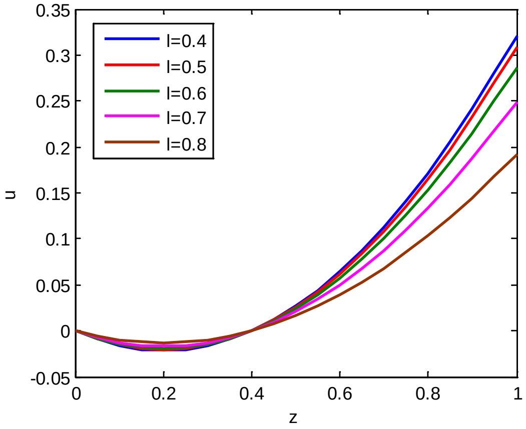


Fig. 3. variation of p on *u* when μ =0.20, l=0.4, h=0.4, p=1, 2, 3, 4, 5

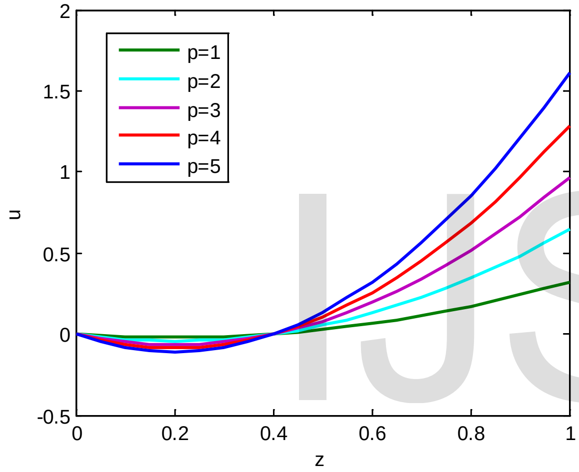


Fig. 4. variation of μ on *u* when l=0.4, p=1, h=0.4, μ= 1.1, 1.2, 1.3, 1.4, 1.5

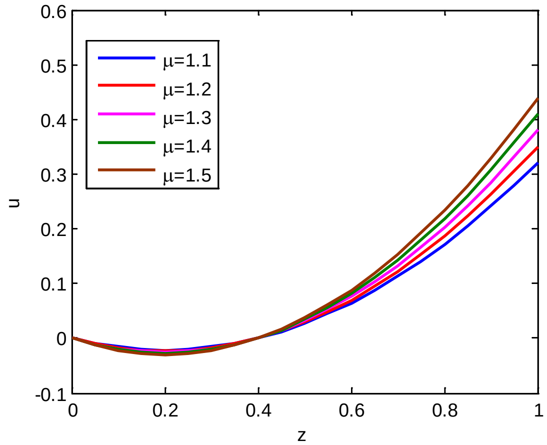


Fig. 5. Variation of l on f when μ =0.20,p=1, h=0.4, l=0.4, 0.5, 0.6, 0.7, 0.8

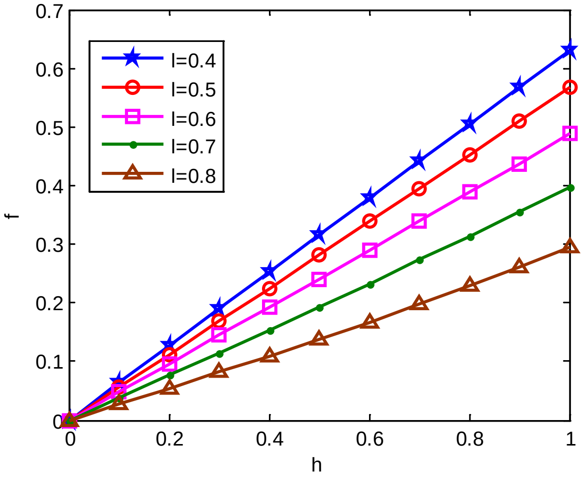
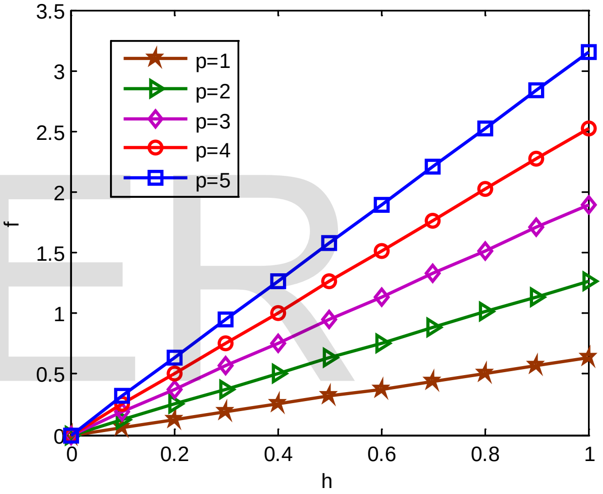


Fig. 6. Variation of p on f when μ =0.20, l=0.4, h=0.4, p=1, 2, 3, 4, 5



5. CONCLUSION

In this paper we studied the effect of the velocity and friction of porous elliptic plates lubricated with couple stress fluid considering the effects of slip velocity. The effect of various parameters like the couple stress parameter, the viscosity, the fluid film height and the pressure gradient revealed that the increase of the parameter decreases the velocity of approach. The value of friction was found to reduce with an increase in the couple stress parameter and the friction decreases with decrease in the pressure gradient. Further efforts are made to study the same with the effect of surface roughness coupled with the porosity of elliptic plate.

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